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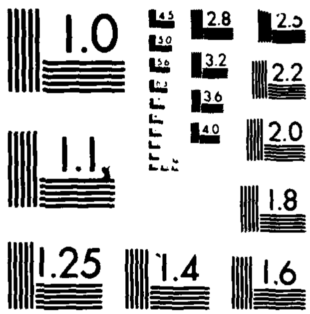

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ON CONTINUOUS DISTRIBUTIONS OF DISLOCATIONS IN NONLOCAL ELASTICITY

A. Cemal Eringen
PRINCETON UNIVERSITY

Technical Report No. 60
Civil Engng. Res. Rep. No. 84-SM-1

PRINCETON UNIVERSITY
Department of Civil Engineering

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ON CONTINUOUS DISTRIBUTIONS OF DISLOCATIONS IN NONLOCAL ELASTICITY

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ABSTRACT

A linear, nonlocal continuum theory of dislocations is developed. The field equations are given for the dislocation density and the stress fields due to continuous distribution of dislocations. Green's functions are obtained for two and three-dimensional media and an integral formula is given for line distribution of dislocations generalizing Peach-Koehler formula of the classical (local) theory. Unlike the classical theory, no stress singularities occur so that self-stress and energies of dislocation loops can be calculated involving no divergences. Exact solutions given for the line and circular distributions of dislocations verify these expectations.

1. INTRODUCTION

In classical (local) theory, the displacement and stress fields due to a continuous distribution of dislocations can be calculated by means of various surface and volume integrals once the Green's functions for the dislocation density is known. To obtain the Greens function one solves partial differential equations for the stress functions. For example, the stress due to a line distribution of dislocation is given by the Peach-Koehler formula. One of the basic difficulties of this theory is that the self-stress and energies of dislocation loops possess mathematical singularities so that calculations will have to be cut-off near the lines of dislocation or the core region.

In several previous papers (cf. 1-4), I have shown that the solutions for the single dislocation involve no stress or energy singularities at the core regions. Moreover, calculated theoretical strengths of solids and dispersion curves for plane waves agree quite well with those known from the atomic lattice dynamics and/or experiments. Therefore, it is expected that the nonlocal theory will eliminate the classical singularities for the self stress and energies of dislocation loops. The raison d'être of the present paper stems from the need to develop a theory of continuous distribution of dislocations based on the nonlocal elasticity, that can hopefully predict the physical phenomena in the microscopic and atomic scales where classical theory fails to apply.

In Section 2, I summarize the basic equations of the nonlocal theory of linear isotropic elastic solids. In Section 3, I develop the field equations for the continuous distribution of dislocations. Green tensors for the Beltrami and Airy stress functions for solids of infinite extends are obtained in Section 4. The stress fields are given by a volume integral generalizing celebrated Peach-Koehler formula of the classical theory. In Section 5, I derive explicit expressions of the stress fields for the uniform distributions of screw dislocations along a straight line segment and along a circular loop. The exact formulas obtained for these cases contain no singularities, justifying our expectation. Section 6 contains calculations of stress fields and a discussion of the reduction of yield stress with the dislocation pile-up.

The simplicity and the aesthetics of these results, I believe, justifiably indicate the power and the potential of the nonlocal theory in the treatment of the physical phenomena with characteristic lengths in the microscopic and atomic scales.

2. BASIC EQUATIONS

The linear theory of nonlocal elasticity is based on Cauchy's equations of motion

$$t_{k\ell,k} + \rho(f_\ell - \ddot{u}_\ell) = 0 \tag{2.1}$$

and the integral constitutive equations

$$t_{k\ell}(\underset{\sim}{x},t) = \int_V c_{k\ell mn}(\underset{\sim}{x}'-\underset{\sim}{x})\, e_{mn}(\underset{\sim}{x}')\, dv(\underset{\sim}{x}') \tag{2.2}$$

where $t_{k\ell}$, ρ, f_ℓ, u_ℓ and $e_{k\ell}$ are respectively, the stress tensor, the mass density, the body force density, the displacement vector and the linear strain tensor defined by

(2.3) $$e_{k\ell} = \frac{1}{2}(u_{k,\ell} + u_{\ell,k})$$

In (2.2), $c_{k\ell mn}$ is a function of the vector $\underset{\sim}{x}'-\underset{\sim}{x}$ and the integral is over the volume of the entire body. Consequently, the stress at a point $\underset{\sim}{x}$ depends on the strain at all points $\underset{\sim}{x}'$ of the body.

Throughout this paper, we employ rectangular coordinates x_k, $k=1,2,3$, and use the usual summation convention on repeated indices. Also a superposed dot indicates the time rate and an index followed by a comma partial differentiation with respect to x_k, e.g.

$$\dot{u}_k = \frac{\partial u_k}{\partial t}, \qquad u_{k,\ell} = \frac{\partial u_k}{\partial x_\ell}$$

The kernel $c_{k\ell mn}$ possesses certain symmetry regulations and it depends on a length scale. For isotropic solids (2.2) takes the simple form

(2.4) $$t_{k\ell}(\underset{\sim}{x},t) = \int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\,\sigma_{k\ell}(\underset{\sim}{x}')\,dv(\underset{\sim}{x}')$$

where $\sigma_{k\ell}$ is the classical (local) stress tensor given by the Hooke's law

(2.5) $$\sigma_{k\ell} = \lambda\, e_{rr}\delta_{k\ell} + 2\mu\, e_{k\ell}$$

and α is a function of the distance $|\underset{\sim}{x}'-\underset{\sim}{x}|$. It also depends on a length scale ε that may be taken to be proportional to an internal characteristic length a

(2.6) $$\varepsilon = e_0\, a$$

where e_0 is a non-dimensional material property which may be determined by one experiment or comparison with calculations based on lattice dynamics[3-5]. The internal characteristic length a may be taken as the lattice parameter for single crystals, granular distance for amorphous materials, and the average distance for fiber composites. As $\varepsilon \to 0$, $t_{k\ell} \to \sigma_{k\ell}$ and (2.4) reduces to Hooke's law $t_{k\ell} = \sigma_{k\ell}$. Thus, $\alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ is a Dirac delta sequence.

In several previous papers[3,5], I have discussed the properties of $\alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ and gave representations which lead to excellent agreement with known atomic calculations on dispersions of waves[3,6] in the entire Brillouin zone and on theoretical strengths of solids.[4] For example, for the two-dimensional case, an appropriate kernel is

$$\alpha(|\underset{\sim}{x}|,\varepsilon) = (2\pi\varepsilon^2)^{-1} K_0(\sqrt{\underset{\sim}{x}\cdot\underset{\sim}{x}}/\varepsilon) \tag{2.7}$$

which satisfies the equation

$$(1-\varepsilon^2\nabla^2)\alpha = \delta(|\underset{\sim}{x}'-\underset{\sim}{x}|) \tag{2.8}$$

vanishing at infinity. In fact, for the infinite solid, it can be shown that α is the Green's function satisfying (2.8) in three-dimensions also.

Using (2.8) in (2.4), we obtain

$$(1-\varepsilon^2\nabla^2)t_{k\ell} = \sigma_{k\ell} \tag{2.9}$$

By means of (2.1) and (2.9), we then find that

$$(\lambda+\mu)u_{k,k\ell} + \mu\, v_{\ell,kk} + (1-\varepsilon^2\nabla^2)(\rho f_\ell - \rho\ddot{u}_\ell) = 0 \tag{2.10}$$

These are the partial differential equations for u_k, replacing Navier's equations of classical elasticity. For the static case and vanishing body forces, this reduces classical Navier's equation

$$(2.11) \qquad (\lambda+\mu)\, u_{k,k\ell} + \mu\, u_{\ell,kk} = 0$$

However, note that the stress field is determined by solving (2.9) under appropriate boundary conditions.

3. CONTINUOUS DISTRIBUTION OF DISLOCATIONS

Continuous distribution of dislocations is envisaged as follows: A small neighborhood $n(\underset{\sim}{x})$ of $\underset{\sim}{x}$ in a distorted body of volume V, may be relaxed to a small neighborhood $N(\underset{\sim}{X})$ of the image of $\underset{\sim}{X}$ of $\underset{\sim}{x}$, in an undistorted (or a natural) configuration V, by releasing constraints exerted to $n(\underset{\sim}{x})$ by the rest of the body. A line element $d\underset{\sim}{x}$ at $\underset{\sim}{x} \in n(\underset{\sim}{x})$ can be expressed in terms of its image $d\underset{\sim}{X} \in N(X)$ by

$$(3.1) \qquad d\underset{\sim}{x} = \underset{\sim}{A}\, d\underset{\sim}{x}$$

where $\underset{\sim}{A}(\underset{\sim}{X})$ is called the elastic distortion. It is assumed that $\underset{\sim}{A}(\underset{\sim}{X})$ is continuously differentiable and possesses uniqie inverse so that

$$(3.2) \qquad d\underset{\sim}{X} = \underset{\sim}{A}^{-1}\, d\underset{\sim}{x}$$

Consider a smooth surface S in V bounded by a closed curve C. The true Burger's vector $\underset{\sim}{b}$ of the dislocations piercing through S is defined by

(3.3) $$\underset{\sim}{b} = \oint_C d\underset{\sim}{X} = \oint_C \overset{-1}{\underset{\sim}{A}} \, d\underset{\sim}{x} = \int_S \underset{\sim}{a}\,\underset{\sim}{n}\,ds$$

where $\underset{\sim}{n}$ is the unit normal to S, the positive sense of C being counter-clockwise when sighting along $\underset{\sim}{n}$. Here $\underset{\sim}{a}$ is called the true dislocation density

(3.4) $$\underset{\sim}{a} = \text{curl}\, \overset{-1}{\underset{\sim}{A}} \qquad \text{or} \qquad a_{jk} = \varepsilon_{kmn} \overset{-1}{A}_{jn,m}$$

For small distortions, we can write

(3.5) $$A_{k\ell} = \delta_{k\ell} + \alpha_{k\ell}\,, \qquad \overset{-1}{A}_{k\ell} = \delta_{k\ell} - \alpha_{k\ell}$$

so that

(3.6) $$a_{jk} = \varepsilon_{kmn}\, \alpha_{jm,n}$$

From this, it follows that

(3.7) $$a_{jk,k} = 0$$

The linear strain tensor $e_{k\ell}$ and rotation tensor $r_{k\ell}$ are given by

$$(3.8) \qquad e_{k\ell} = \frac{1}{2}(\alpha_{k\ell} + \alpha_{\ell k}) , \qquad r_{k\ell} = \frac{1}{2}(\alpha_{k\ell} - \alpha_{\ell,k})$$

The strain incompatibility is expressed by

$$(3.9) \qquad \varepsilon_{ijk}\,\varepsilon_{\ell mn}\,e_{in,jm} = \eta_{k\ell}$$

where $\eta_{k\ell}$ is called the incompatibility tensor and is given by

$$(3.10) \qquad \eta_{k\ell} = \frac{1}{2}(\varepsilon_{kmn}a_{n\ell,m} + \varepsilon_{\ell mn}a_{nk,m})$$

All these results are well-known in classical theory (cf. Ref. [7]).

In nonlocal elasticity, the strain tensor can be solved by using (2.9) and (?.5)

$$(3.11) \qquad e_{k\ell} = \frac{1}{2\mu}(1 - \varepsilon^2\nabla^2)(t_{k\ell} - \frac{\nu}{1+\nu}\ t_{rr}\delta_{k\ell})$$

where $\nu = \lambda/2(\lambda+\mu)$ is the Poisson's ratio. Substituting (3.11) into (3.9), we obtain

$$(3.12) \qquad (1 - \varepsilon^2\nabla^2)[\nabla^2 t_{k\ell} + \frac{1}{1+\nu}(t_{rr,k\ell} - \nabla^2 t_{rr}\delta_{k\ell})] = 2\mu\,\eta_{k\ell}$$

These equations must be solved under the conditions of equilibrium

$$(3.13) \qquad t_{k\ell,k} = 0$$

Following Kröner's approach[8], modifying the Beltrami solution of (3.13), we take

$$t_{k\ell}/2\mu = \nabla^2\chi_{k\ell} + \frac{1}{1-\nu}(\chi_{rr,k\ell} - \nabla^2\chi_{rr}\,\delta_{k\ell}) \tag{3.14}$$

where the symmetric stress function $\chi_{k\ell}$ is subject to

$$\chi_{k\ell,\ell} = 0 \,. \tag{3.15}$$

Substituting (3.14) into (3.12), we obtain

$$(1 - \varepsilon^2\nabla^2)\,\nabla^4\chi_{k\ell} = \eta_{k\ell} \tag{3.16}$$

Thus, given the dislocation density function $a_{k\ell}$, through (3.10), we calculate $\eta_{k\ell}$. The solution of (3.16) gives $\chi_{k\ell}$ and (3.14) the stress field.

Equation (3.16) is <u>singularly perturbed</u> and as expected in the limit $\varepsilon \to 0$, (3.16) reduces to the classical equation for $\chi_{k\ell}$.

To obtain the solution of (3.16), we must find the Green's Tensor $G_{k\ell mn}(\underset{\sim}{x},\underset{\sim}{\xi})$ which satisfies

$$(1 - \varepsilon^2\nabla^2)\,\nabla^4 G_{k\ell mn} = \delta(\underset{\sim}{x}-\underset{\sim}{\xi})\,\delta_{k\ell}\delta_{mn} \,. \tag{3.17}$$

The solution of (3.16) is then given by

$$\chi_{k\ell} = \int_V G_{k\ell mn}(\underset{\sim}{x},\underset{\sim}{\xi})\,\eta_{mn}(\underset{\sim}{\xi})\,dv(\underset{\sim}{\xi}) \tag{3.18}$$

subject to supplementary conditions (3.15).

4. GREEN'S TENSORS AND STRESS FIELDS

Here we determine Green's tensors for two and three-dimensional bodies of infinite extends.

(i) Three-Dimensional Infinite Space

The operator ∇^2 is invariant under rotations of coordinates. For the infinite space, we look for a solution of (3.17) which depends on $|\underset{\sim}{x}-\underset{\sim}{\xi}|$ only, i.e.,

$$(1 - \varepsilon^2\nabla^2)\, \nabla^4 G = \delta(\underset{\sim}{x} - \underset{\sim}{\xi}) \tag{4.1}$$

Since the operators $1-\varepsilon^2\nabla^2$ and ∇^4 are commutative, we set

$$\begin{aligned} (1 - \varepsilon^2\nabla^2)\, G &= H\,, \\ \nabla^4 H &= \delta(|\underset{\sim}{x}-\underset{\sim}{\xi}|) \end{aligned} \tag{4.2}$$

For the infinite space, H is given by

$$H = -\,|\underset{\sim}{x} - \underset{\sim}{\xi}|/8\pi \tag{4.3}$$

In spherical coordinates using

$$\nabla^2 = \frac{1}{r^2}\,(r^2 \frac{d}{dr})$$

we obtain for G :

$$(4.4) \qquad G(|\underset{\sim}{x}-\underset{\sim}{\xi}|) = \frac{\varepsilon^2 G_0}{4\pi|\underset{\sim}{x}-\underset{\sim}{\xi}|} \exp(-|\underset{\sim}{x}-\underset{\sim}{\xi}|/\varepsilon) - \frac{|\underset{\sim}{x}-\underset{\sim}{\xi}|}{8\pi}, \qquad \varepsilon \neq 0$$

$$(4.5) \qquad G(|\underset{\sim}{x}-\underset{\sim}{\xi}|) = -\frac{|\underset{\sim}{x}-\underset{\sim}{\xi}|}{8\pi}, \qquad \varepsilon = 0$$

Here G_0 is an arbitrary constant which may be chosen $G_0 = 1$ to render t_{kk} regular at $\underset{\sim}{x} = \underset{\sim}{\xi}$. The solution of (3.16) for the infinite solid is given by

$$(4.6) \qquad \chi_{k\ell} = \int_V G(|\underset{\sim}{x}-\underset{\sim}{\xi}|)\, \eta_{k\ell}(\underset{\sim}{\xi})\, dv(\underset{\sim}{\xi})$$

which satisfies the conditions (3.15) on account of (3.7) and (3.10). If we substitute from (3.10), this gives

$$(4.7) \qquad \chi_{k\ell}(\underset{\sim}{x}) = \frac{1}{2}\varepsilon_{ijk} \int_V a_{j\ell}(\underset{\sim}{\xi}) \frac{\partial G}{\partial x_i} dv(\underset{\sim}{\xi}) + \frac{1}{2}\varepsilon_{ij\ell} \int_V a_{jk}(\underset{\sim}{\xi}) \frac{\partial G}{\partial x_i} dv(\underset{\sim}{\xi})$$

where we used the Green-Gauss theorem and set a surface term at infinity to zero.

From (4.7), one can obtain various special cases involving surface and line distributions of dislocations. For example, for a line distribution of dislocation along a closed curve C, we obtain

$$\chi_{k\ell} = \frac{1}{2}\,\varepsilon_{kij} b_j \oint_C \frac{\partial G}{\partial x_i}\, d\ell_\ell + \frac{1}{2}\,\varepsilon_{\ell ij} b_j \oint_C \frac{\partial G}{\partial x_i}\, d\ell_k \tag{4.8}$$

where b_j is the Burger's vector per unit length of C and $d\ell_i$ is the element of the arc.

Upon substituting (4.8) into (3.14), we obtain the stress field due to a line distribution of dislocations

$$\begin{aligned} t_{k\ell}/2\mu = \frac{1}{2}\,\varepsilon_{rij} b_j \oint_C \; & [\nabla^2 G_{,i}(\delta_{rk} d\ell_\ell + \delta_{r\ell} d\ell_k) \\ & + \frac{2}{1-\nu}(G_{,k\ell i} - \nabla^2 G_{,i}\,\delta_{k\ell}) d\ell_r] \end{aligned} \tag{4.9}$$

This result is identical to the Peach and Koehler[7] formula with modification that here G is the nonlocal Green's function (4.4) with $\varepsilon \neq 0$. As we shall see, the most interesting new feature of (4.9) is that, with G given by (4.4), at a point on the dislocation line C, the stress is finite so that the self-stress and energies of dislocation loops can be calculated, free of infinities.

(ii) Two-Dimensional Infinite Plane

In the case of the plane strain, introducing the Airy's stress function $\Phi(x_1, x_2)$ by

$$t_{11} = \Phi_{,22}\,, \qquad t_{22} = \Phi_{,11}\,, \qquad t_{12} = -\Phi_{,12} \tag{4.10}$$

we obtain an equation replacing (3.16)

$$(4.11) \qquad (1 - \varepsilon^2\nabla^2)\, \nabla^4\Phi = 2\mu\,\eta$$

where

$$(4.12) \qquad \eta = \eta_{33} = a_{23,1} - a_{13,2}$$

$$a_{23} = \alpha_{21,2} - \alpha_{22,1}, \qquad a_{13} = \alpha_{11,2} - \alpha_{12,1}$$

depend on x_1 and x_2 only.

Green's function in this case, can be found similar to decomposition (4.2) with ∇^2 given by

$$\nabla^2 = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)$$

Hence,

$$(4.13) \qquad G(|\underset{\sim}{x}-\underset{\sim}{\xi}|) = \frac{G_0}{2\pi} K_0(|\underset{\sim}{x}-\underset{\sim}{\xi}|/\varepsilon) - \frac{(\underset{\sim}{x}-\underset{\sim}{\xi})\cdot(\underset{\sim}{x}-\underset{\sim}{\xi})}{8\pi\varepsilon^2}\, \ell n(|\underset{\sim}{x}-\underset{\sim}{\xi}|/\varepsilon), \quad \varepsilon \neq 0$$

$$(4.14) \qquad G(|\underset{\sim}{x}-\underset{\sim}{\xi}|) = -\frac{(\underset{\sim}{x}-\underset{\sim}{\xi})\cdot(\underset{\sim}{x}-\underset{\sim}{\xi})}{8\pi}\, \ell n(|\underset{\sim}{x}-\underset{\sim}{\xi}|), \qquad \varepsilon = 0$$

where $K_0(z)$ is the modified Bessel's function. Again, we take $G_0 = 1$ to render t_{kk} regular at $\underset{\sim}{x} = \underset{\sim}{\xi}$.

Airy's stress function is obtained to be

$$(4.15) \qquad \Phi(\underset{\sim}{x}) = 2\mu \int_S [G_{,1}\, a_{23}(\underset{\sim}{\xi}) - G_{,2}\, a_{13}(\underset{\sim}{\xi})]\, ds$$

where we used the Green-Gauss theorem and set a line integral to zero at infinity. For a <u>line</u> distribution of dislocations in the $x_3 = 0$ - plane, we obtain

$$(4.16) \qquad \Phi(\underset{\sim}{x}) = -2\mu \int_C [G_{,1}\, b_2(\underset{\sim}{\xi})\, d\xi_1 + G_{,2}\, b_1(\underset{\sim}{\xi})\, d\xi_2]$$

The stress field follows from (4.10)

$$(4.17) \qquad \begin{aligned} t_{11} &= -2\mu \int_C (G_{,122}\, b_2\, d\xi_1 + G_{,222}\, b_1\, d\xi_2)\,, \\ t_{22} &= -2\mu \int_C (G_{,111}\, b_2\, d\xi_1 + G_{,211}\, b_1\, d\xi_2)\,, \\ t_{12} &= 2\mu \int_C (G_{,112}\, b_2\, d\xi_1 + G_{,212}\, b_1\, d\xi_2) \end{aligned}$$

(iii) <u>Anti-Plane Strain</u>

In the case of anti-plane strain, equations of equilibrium are satisfied if

$$(4.18) \qquad t_{13} = \frac{\partial\phi}{\partial x_2}\,, \qquad t_{23} = -\frac{\partial\phi}{\partial x_1}$$

and we obtain

$$(4.19) \qquad (1 - \varepsilon^2\nabla^2)\, \nabla^2\phi = \mu\, a_{33}$$

where

$$(4.20) \qquad a_{33} = \alpha_{31,2} - \alpha_{32,1}$$

Green's function for this case, is obtained to be

$$(4.21) \qquad G(|\underset{\sim}{x}-\underset{\sim}{\xi}|) = -\frac{1}{2\pi}\left[\ell n(|\underset{\sim}{x}-\underset{\sim}{\xi}|/\varepsilon) + K_0(|x-\xi|/\varepsilon)\right], \qquad \varepsilon \neq 0$$

$$(4.22) \qquad G(|\underset{\sim}{x}-\underset{\sim}{\xi}|) = -\frac{1}{2\pi}\, \ell n(|\underset{\sim}{x}-\underset{\sim}{\xi}|) \qquad \varepsilon = 0$$

The stress field is given by

$$(4.23) \qquad t_{13} = \mu \int_S G_{,2}\, b(\underset{\sim}{\xi})\, d\xi_1\, d\xi_2$$

$$t_{23} = -\mu \int_S G_{,1}\, b(\underset{\sim}{\xi})\, d\xi_1\, d\xi_2$$

For a line distribution of dislocations on the plane $x_3 = 0$, we have

$$(4.24) \qquad t_{13} = \mu \int_C G_{,2}\, b(\underset{\sim}{\xi})\, d\ell\,, \qquad t_{23} = -\mu \int_C G_{,1}\, b(\underset{\sim}{\xi})\, d\ell$$

In plane polar coordinates (r,θ), the stress field is given by

(4.25)

$$t_{zr} = \mu \int_C \frac{1}{r} \frac{\partial G}{\partial \theta} B(\underset{\sim}{\xi})\, d\ell$$

$$t_{z\theta} = -\mu \int_C \frac{\partial G}{\partial r} b(\underset{\sim}{\xi})\, d\ell$$

5. UNIFORM DISTRIBUTIONS OF SCREWS

Here we calculate the stress field for two different uniform line distributions of screw dislocations:

(i) *Screw Dislocation Along a Straight Line*

Consider a line distribution of screw dislocations of constant Burger's vector along a straight line segment $|x_1| \leq \ell$, $x_2 = x_3 = 0$. Green's function is given by

(5.1) $$G(|\underset{\sim}{x} - \underset{\sim}{\xi}|) = -\frac{1}{2\pi}[\ln(\rho/\varepsilon) + K_0(\rho/\varepsilon)]$$

where

(5.2) $$\rho = [(x_1 - \xi)^2 + x_2{}^2]^{\frac{1}{2}}$$

We can evaluate t_{23} given by (4.24) immediately since

$$G,_1 \equiv \partial G/\partial x_1 = -\partial G/\partial \xi$$

and we have

(5.3) $$t_{23} = \mu b\ [\ G(|\ell - \xi|) - G(|\ell + \xi|)\ ]$$

Explicitly,

(5.4) $$t_{23} = -\frac{\mu b}{2\pi}\left\{ \ell n\left[\frac{(x_1-\ell)^2+x_2^2}{(x_1+\ell)^2+x_2^2}\right]^{\frac{1}{2}} + K_0\left[\sqrt{(x_1-\ell)^2+x_2^2}/\varepsilon\right] - K_0\left[\sqrt{(x_1+\ell)^2+x_2^2}/\varepsilon\right]\right.$$

Along the line of screws, this gives

(5.5) $$t_{23}(x_1,0) = -\frac{\mu b}{2\pi}\left\{ \ell n\left(\frac{|x_1-\ell|}{|x_1+\ell|}\right) + K_0(|x_1-\ell|/\varepsilon) - K_0(|x_1+\ell|/\varepsilon)\right\}$$

Calculation of t_{13} is complicated. However, it is easy to see that

(5.6) $$t_{13}(x_1,0) = 0$$

Unlike the classical case, $t_{23}(x_1,0)$ has no singularity at the end points $x_1 = \pm\ell$ of the screw line. In fact, we have

(5.7) $$t_{23}(\pm\ell,0) = \pm\frac{\mu}{2\pi}[\ell n(\ell/\varepsilon) + K_0(2\ell/\varepsilon)]$$

for $\ell/\varepsilon >> 1$, we have the asymptotic value

(5.8) $$t_{23}(\pm\ell,0) \to \pm\frac{\mu b}{2\pi}[\ell n(\ell/\varepsilon) + (\pi\varepsilon/4\ell)^{\frac{1}{2}}\exp(-2\ell/\varepsilon)]$$

where the second term can also be neglected as compared to the first one for large ℓ/ε.

(ii) *Uniform Distribution of Screw Dislocations Along a Circle*

Suppose that in the plane $x_3 = 0$, there is a uniformly distributed screw dislocations along a circle of radius R. In plane polar coordinates, we have

$$(5.9)\qquad \begin{aligned} &x_1 = r\cos\theta\,, \qquad x_2 = r\sin\theta\,,\\ &\xi_1 = R\cos\phi\,, \qquad \xi_2 = R\sin\phi\,,\\ &|\underset{\sim}{x} - \underset{\sim}{\xi}| = [\,r^2 + R^2 - 2rR\cos(\phi-\theta)\,]^{\frac{1}{2}} \end{aligned}$$

Greens' function is given by

$$(5.10)\qquad G(|\underset{\sim}{x} - \underset{\sim}{\xi}|) = -\frac{1}{2\pi}\,[\ell n(|\underset{\sim}{x} - \underset{\sim}{\xi}|/\varepsilon) + K_o(|\underset{\sim}{x} - \underset{\sim}{\xi}|/\varepsilon)]$$

To calculate the stress field t_{zr} and $t_{z\theta}$ we must evaluate two integrals

$$(5.11)\qquad I_1 = \int_{\theta}^{2\pi+\theta} \ell n\left\{\frac{1}{\varepsilon}\,[r^2 + R^2 - 2\,r\,R\cos(\phi-\theta)]^{\frac{1}{2}}\right\} d\phi$$

$$(5.12)\qquad I_2 = \int_{\theta}^{2\pi+\theta} K_o\left\{\frac{1}{\varepsilon}\,[r^2 + R^2 - 2\,r\,R\cos(\phi-\theta)\,]^{\frac{1}{2}}\right\} d\phi$$

To evaluate (5.11), we write

$$(5.13)\qquad \ln\left\{\frac{1}{\varepsilon}\left[r^2+R^2-2rR\cos(\phi-\theta)\right]^{\frac{1}{2}}\right\} = \ln\left(\frac{|r+R|}{\varepsilon}\right) + \ln\left(1-\alpha_o\cos^2\frac{\psi}{2}\right)^{\frac{1}{2}}$$

where

$$(5.14)\qquad \alpha_o = 4rR/(r+R)^2 \quad , \quad \phi-\theta=\psi$$

The second term in (5.13) can be integrated by writing $x=\cos(\psi/2)$ and consulting Ref. 9, p. 562, No. 38. Consequently

$$(5.15)\qquad I_1 = \begin{cases} 2\pi\ \ln(r/\varepsilon) & r>R \\ 2\pi\ \ln(R/\varepsilon) & r<R \end{cases}$$

To evaluate I_2, we employ the integrals 6.684 on p. 741 of Ref. 9, and note that

$$K_o(z) = \frac{\pi i}{2}\left[J_o(iz)+i\,N_o(iz)\right]$$

where J_0 and N_0 are zero order Bessel functions. The result is

$$(5.16)\qquad I_2 = \begin{cases} 2\pi\ I_o(R/\varepsilon)\ K_o(r/\varepsilon) \ , & r>R \\ 2\pi\ I_o(r/\varepsilon)\ K_o(R/\varepsilon) \ , & r<R \end{cases}$$

Consequently,

$$I = \int_0^{2\pi} G(|\underset{\sim}{x}-\underset{\sim}{\xi}|\, d\ell = - \begin{cases} \ell n\ (r/\varepsilon) + I_o(R/\varepsilon)\ K_o(r/\varepsilon), & r>R \\ \ell n\ (R/\varepsilon) + I_o(r/\varepsilon)\ K_o(R/\varepsilon), & r<R \end{cases} \tag{5.17}$$

The stress field is given by

$$t_{zr} = \mu \frac{bR}{r} \frac{\partial I}{\partial \theta} = 0\ , \tag{5.18}$$

$$t_{z\theta} = \mu bR \frac{\partial I}{\partial r} = \begin{cases} \mu\ b \frac{R}{\varepsilon} [\ \frac{\varepsilon}{r} - I_o\ (R/\varepsilon)\ K_1(r/\varepsilon)\ , & r>R \\ \mu\ b \frac{R}{\varepsilon} I_1(r/\varepsilon)\ K_o(R/\varepsilon)\ , & r<R \end{cases}$$

In the special case when $R \to 0$ and $2\pi Rb = b_0$, we obtain

$$t_{z\theta} = \frac{\mu b_o}{2\pi r}\ [\ 1 - \frac{r}{\varepsilon} K_1(r/\varepsilon)\ ] \tag{5.19}$$

which is identical to our previous result for a single screw dislocation having Burger's fector b_0.

6. STRESS DISTRIBUTIONS

Here, I present some numerical results on the stress distributions for the cases discussed in Section 5 and establishes a fracture criteria based on the maximum shear stress.

(i) Single Screw: The shear stress given by (5.19) may be expressed in non-dimensional form:

$$T_\theta(\rho) = (2\pi\varepsilon/\mu b)\, t_{z\theta} = \rho^{-1}[1 - \rho K_1(\rho)] \tag{6.1}$$

where

$$\rho = r/\varepsilon \tag{6.2}$$

The stress field given by (6.1) is displayed graphically in Fig. 1. It has no singularity at $\rho = 0$. In fact, $T_\theta(\rho)$ vanishes at $\rho = 0$ in contradiction to the classical elasticity solution which gives infinite stress at $\rho = 0$. The maximum stress occurs at $\rho = 1.1$ and is given by

$$t_{z\theta\,\max} = 0.3993\,\frac{b}{2\pi\varepsilon} \tag{6.3}$$

If we write $h = \varepsilon/0.3993$, this agrees with Frenkel's estimate of the theoretical strength of single crystals, based on atomic considerations (cf., Kelly [10], p. 12). In fact, if we use $\varepsilon = e_0 a = 0.39\ a$, which is obtained on the basis of matching of the dispersion curve predicted by non-local elasticity and the Born-Kármán lattice model[3], we find for the single aluminum crystal

(6.4) $$t^c/\mu = 0.12 \qquad \{Al: [111] < 1\bar{1}0 > \}$$

This is very close to the theoretical strength $t_y/\mu = 0.11$ based on atomic models.

(ii) <u>Screw Dislocations Along a Straight Line Segment</u>: Even single crystals contain many dislocations. For a uniform distribution of screws along a line segment $|x_1| < \ell$, $x_2 = x_3 = 0$, the shear stress given by (5.5) may be written in non-dimensional form

(6.5) $$T_2 = t_{23}/t_d = \ell n \frac{|x+1|}{|x-1|} + K_0(\gamma|x+1|) - K_0(\gamma|x-1|)$$

where

(6.6) $$t_d = \mu b/2\pi = \mu b_0 N/2\pi\ell \quad , \qquad x = x_1/\ell \quad , \qquad \gamma = \ell/\varepsilon$$

Here b_0 is the atomic Burger's vector and N is the total number of dislocations over a distance ℓ .

The distribution of the shear stress (6.5) as a function of x is shown in Fig. 2 for various values of γ . Behavior of T_2 is governed basically by the first term in (6.5) except near $x = 1$. At $x = 1$, we have

(6.7) $$t_{21}(1) = \frac{\mu b_o N}{2\pi\varepsilon} \frac{\ell n \gamma}{\gamma}$$

The value of $T_2(1)$ is very close to the maximum stress for $\gamma \geq 3$

(cf. Table 1). For a single atomic dislocation according to (6.3), we have the theoretical strength

$$t_y^c = 0.3993 \frac{\mu b_0}{2\pi\varepsilon} \tag{6.8}$$

Combining (6.7) and (6.8), we obtain

$$t_y^d / t_y^c = \frac{N}{0.3993} \frac{\ell n\, \gamma}{\gamma} \tag{6.9}$$

where we set $t_{23}(1) = t_y^d$ = the yield stress for the distributed dislocations. This gives the shear stress reduction due to the presence of $2N$ dislocations distributed uniformly along a straight line segment of length 2ℓ . Since $t_y^d \leq t_y^c$, the maximum number of dislocations is given by

$$N_{max} = 0.3993 \frac{\gamma}{\ell n\, \gamma} \tag{6.10}$$

For $\gamma = 4.02 \times 10^4$, this gives $N_{max} = 1514$, which may be conservative since the distribution is not generally uniform but in an inverse pile-up configuration[11].

(iii) Uniform Distribution of Screws Along a Circle: In this case, the stress fields given by (5.18) may be expressed in non-dimensional form

$$T = t_{z\theta}/\mu b = \begin{cases} \frac{1}{\rho} - \kappa\, I_0(\kappa)\, K_1(\kappa\rho) \ , & \rho > 1 \\ \kappa\, K_0(\kappa)\, I_1(\kappa\rho) \ , & \rho < 1 \end{cases} \tag{6.11}$$

where

(6.12) $$\rho = r/R \ , \qquad \kappa = R/\varepsilon$$

T as a function of ρ, for various values of κ, is displayed in Fig. 3. For value of $\kappa \geq 50$, the maxima of T occurs near $\rho = 1$. The locations and values of T_{max} are given in Table 2.

If b_0 is the atomic Burger's vector, then

(6.13) $$2\pi R b = N b_0$$

where N is the number of dislocations on the dislocation circle with radius R. Using (6.8), (6.12) and (6.13), we obtain

(6.14) $$t_{z\theta}/t_y^c = \frac{N}{0.3993\,\kappa} T$$

According to Fig. 3, $0.324 \lesssim T_{max} \lesssim 1$. Consequently,

(6.15) $$N/1.2\,\kappa \lesssim t_y^d/t_y^c \lesssim N/0.4\,\kappa$$

For perfect crystals with $\varepsilon = 0.39\,a$, this gives approximately

(6.16) $$0.3\,Na/R \lesssim t_y^d/t_y^c \lesssim Na/R$$

indicating reduction of the yield stress with the presence of large number dislocations uniformly distributed along a circle of radius R.

Table 1 : Maximum Shear Stress and its Location

(Line Segment)

γ =	1	1.5	2	3	5	10
x =	1.446	1.197	1.103	1.039	1.000	1.000
T_{2max} =	0.7478	1.0501	1.3008	1.6851	2.3026	2.9957

Table 2 : Maximum Shear Stress and its Location

(Circle)

κ =	1	2	3	5	10	50
ρ =	1.8	1.5	1.4	1.3	1.2	1.1
T_{max} =	0.3243	0.4836	0.5688	0.6630	0.7688	0.9058

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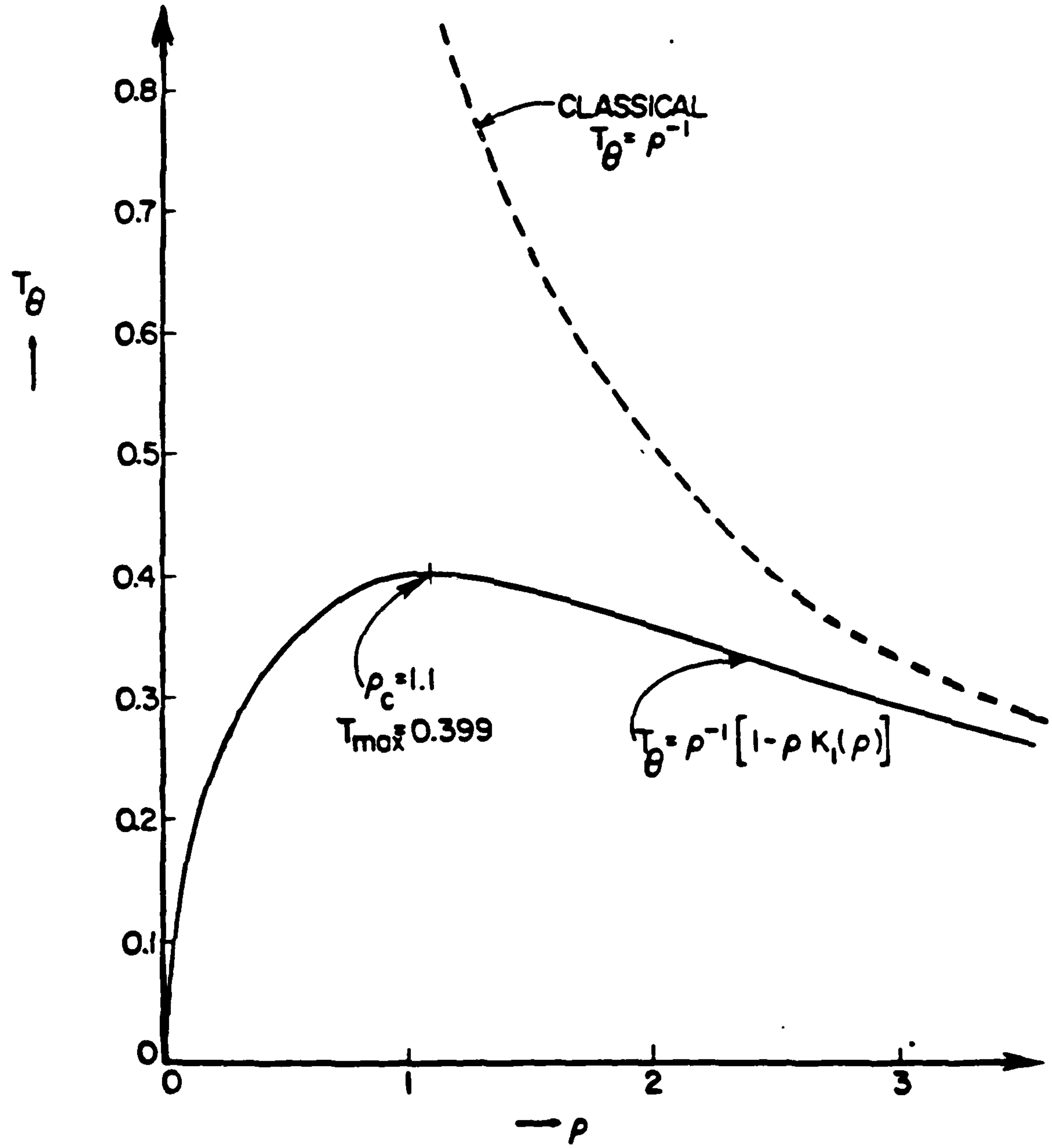


NON-DIMENSIONAL HOOP STRESS FOR SCREW DISLOCATION

FIGURE 1

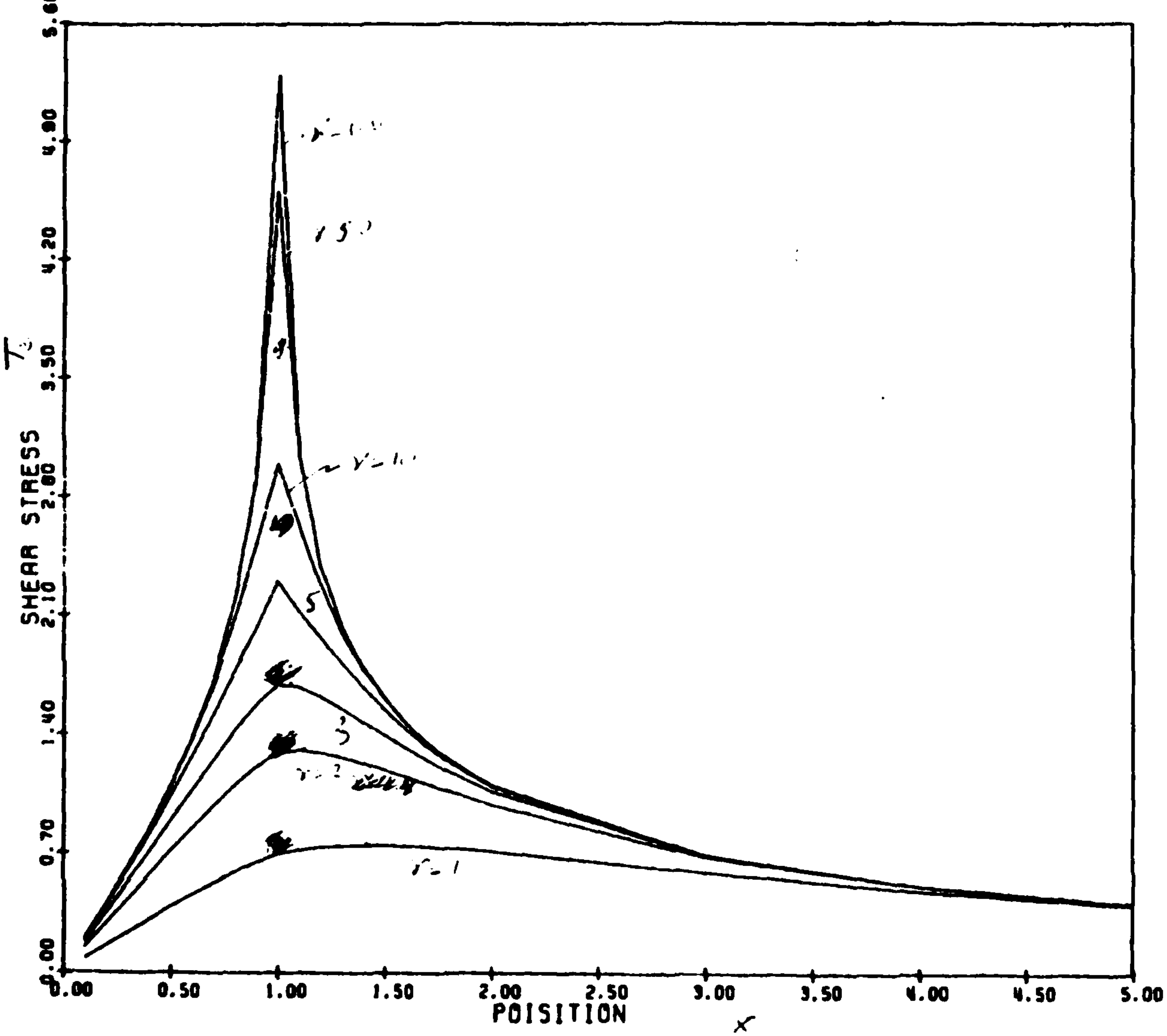
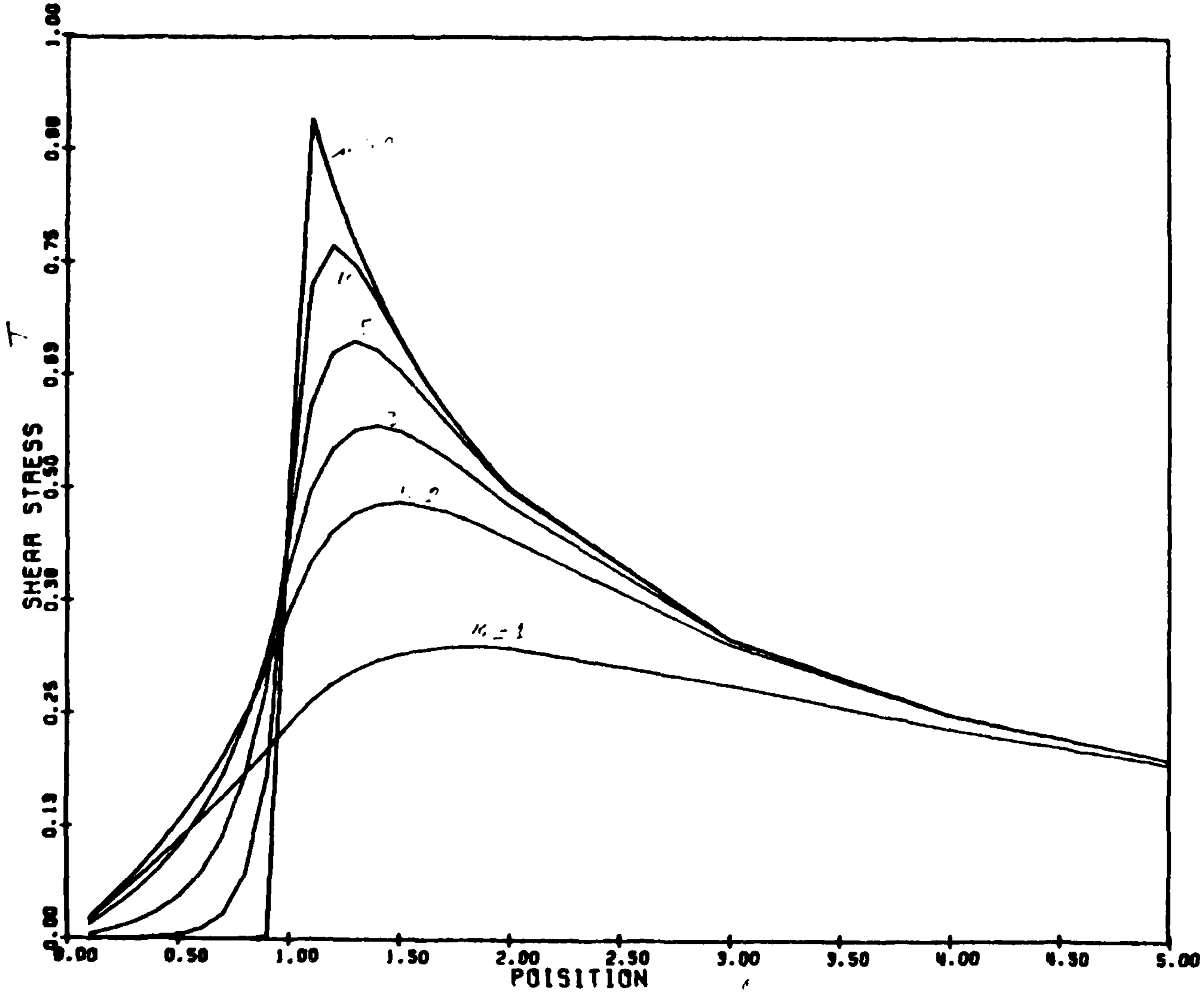


Fig. 2 SHEAR STRESS DISIRIBUTION (Line Segment)



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